Spiral de forme quelconque

Développement excentrique et anisochronisme en position horizontale

Exemple de la spirale d'Archimède

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5 \times 10^{-3}\,mm^2 \quad TOL := 10^{-9}$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L_{sp} = 11.182\,cm \quad \psi_0 := 2 \cdot \pi \cdot n_{sp} \quad \psi_0 = 4.56 \times 10^3\,deg$

Positions du piton $\quad r_P := 0.5 \cdot d2_{sp} \quad \alpha_P := 0 \quad x_P := r_P \cdot cos(\alpha_P) \quad y_P := r_P \cdot sin(\alpha_P)$

$x_P = 2.26\,mm \quad y_P = 0\,mm$

Position du point d'attache à la virole $\quad r_V := 0.5 \cdot d1_{sp} \quad \alpha_V(\theta) := \psi_0 + \theta \quad x_V(\theta) := r_V \cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V \cdot sin(\alpha_V(\theta))$

Forme naturelle du spiral

$$a := \frac{p_{sp}}{2 \cdot \pi} \qquad r_{0s}(\alpha) := r_P - a \cdot \alpha \qquad x_{0s}(\alpha) := r_{0s}(\alpha) \cdot cos(\alpha) \qquad y_{0s}(\alpha) := r_{0s}(\alpha) \cdot sin(\alpha)$$

$$r'(\alpha) := \frac{d}{d\alpha} r_{0s}(\alpha) \qquad d\sigma(\alpha) := \sqrt{r_{0s}(\alpha)^2 + r'(\alpha)^2} \qquad s(\alpha) := \int_0^{\alpha} d\sigma(\alpha')\, d\alpha' \qquad s\left(\frac{\psi_0}{2}\right) = 72.927\,mm$$

$$L := s(\psi_0) \qquad L = 111.835\,mm$$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$I_{33} := I_{f\_rect}(ép, ha) \qquad W_{f3} := W_{f\_rect}(ép, ha) \qquad \sigma_{max} := \frac{E \cdot I_{33}}{L \cdot W_{f3}} \cdot \theta_0 \qquad \sigma_{max} = 132.275 \frac{N}{mm^2}$$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Déformée du spiral avec la virole non liée à l'axe de balancier

$$z_P := x_P + i \cdot y_P \qquad z_0(\alpha) := r_{0s}(\alpha) \cdot exp(i \cdot \alpha)$$

$$z_1(\theta, \alpha) := z_0(\alpha) \cdot exp\left(i \cdot \theta \cdot \frac{s(\alpha)}{L}\right) - i \cdot \frac{\theta}{L} \cdot \int_0^{\alpha} z_0(\alpha') \cdot exp\left(i \cdot \theta \cdot \frac{s(\alpha')}{L}\right) \cdot d\sigma(\alpha')\, d\alpha'$$

Graphe de la déformation

Forme naturelle

$n := 50 \cdot partentière(n_{sp}) + 1 \quad i := 0 .. n-1 \quad \Delta\alpha := \frac{\psi_0}{n-1} \quad \alpha_i := i \cdot \Delta\alpha$

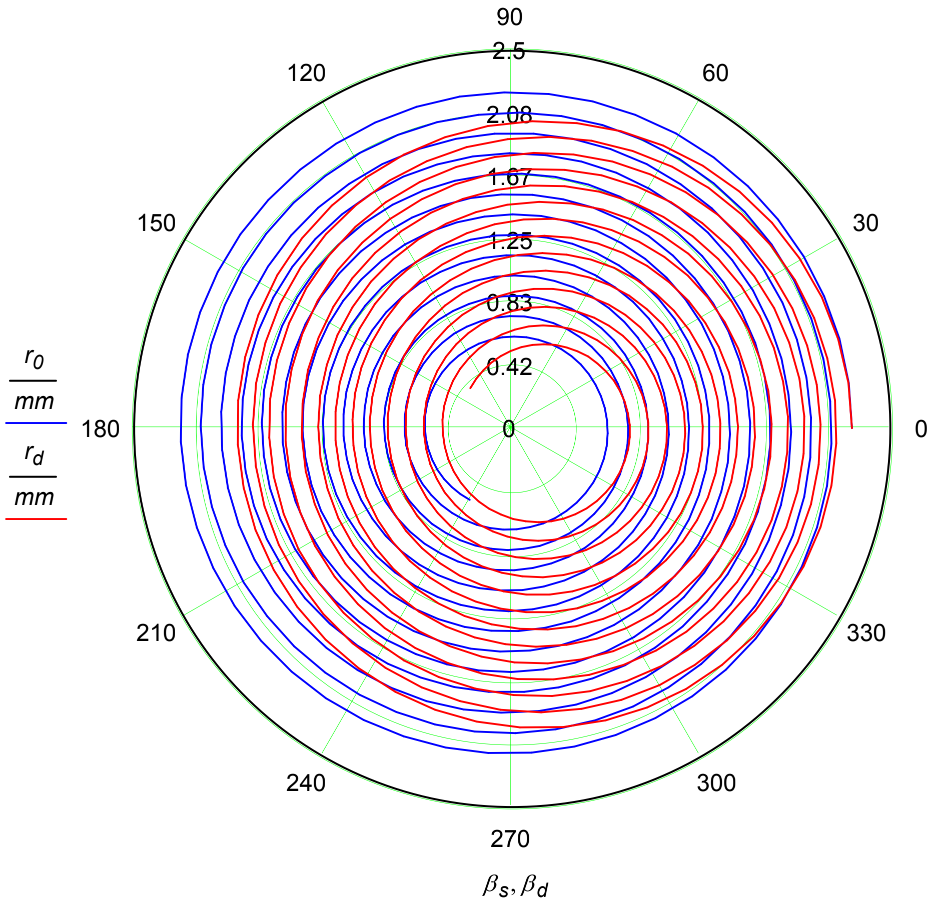
$x_{0_i} := x_{0s}(\alpha_i) \quad y_{0_i} := y_{0s}(\alpha_i) \quad r_0 := \overrightarrow{\sqrt{x_0^2 + y_0^2}} \quad \beta_s := \overrightarrow{Atan(x_0, y_0)}$

Déformée

$z_{d_i} := z_1(\theta_0, \alpha_i) \quad n_{pt} := dernier(z_d) \quad x_d := Re(z_d) \quad y_d := Im(z_d) \quad r_d := \overrightarrow{|z_d|} \quad r_{d_{npt}} = 0.378\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)} \quad \beta_{d_0} = 0\,deg \quad \beta_{d_{npt}} = 135.517\,deg$

$mod(\alpha_V(\theta_0), 2\cdot\pi) = 150\,deg$



$x_V(\theta_0) = -0.476\,mm \quad x_{d_{npt}} - x_V(\theta_0) = 0.207\,mm \quad y_V(\theta_0) = 0.275\,mm \quad y_{d_{npt}} - y_V(\theta_0) = -0.01\,mm$

Déplacement de la virole libre

$$\mathbf{\Delta_1}(\theta) := \frac{i\cdot\theta}{L}\cdot\int_0^{\psi_0} z_0(\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha')}{L}\right)\cdot d\sigma(\alpha')\,d\alpha' \quad u_1(\theta) := Re(\mathbf{\Delta_1}(\theta)) \quad v_1(\theta) := Im(\mathbf{\Delta_1}(\theta))$$

$u_1(\theta_0) = -0.207\,mm \quad v_1(\theta_0) = 0.01\,mm$

Calcul des réactions sur la virole

Première approximation

$$\xi_{0s} := \frac{1}{L} \cdot \int_0^{\psi_0} x_{0s}(\alpha) \cdot d\sigma(\alpha)\, d\alpha \qquad \eta_{0s} := \frac{1}{L} \cdot \int_0^{\psi_0} y_{0s}(\alpha) \cdot d\sigma(\alpha)\, d\alpha$$

$$q2_{0s} := \frac{1}{L} \cdot \int_0^{\psi_0} y_{0s}(\alpha)^2 \cdot d\sigma(\alpha)\, d\alpha \quad p2_{0s} := \frac{1}{L} \cdot \int_0^{\psi_0} x_{0s}(\alpha)^2 \cdot d\sigma(\alpha)\, d\alpha \quad k_{0s} := \frac{1}{L} \cdot \int_0^{\psi_0} x_{0s}(\alpha) \cdot y_{0s}(\alpha) \cdot d\sigma(\alpha)\, d\alpha$$

$$\xi_{0s} = -1.363 \times 10^{-3}\, mm \quad \eta_{0s} = 0.047\, mm \quad q2_{0s} = 1.352\, mm^2 \quad p2_{0s} = 1.353\, mm^2 \quad k_{0s} = 0.026\, mm^2$$

$$\mathbf{S_0} := \frac{L}{E \cdot I_{33}} \cdot \begin{pmatrix} q2_{0s} - \eta_{0s}^2 & \eta_{0s} \cdot \xi_{0s} - k_{0s} \\ \eta_{0s} \cdot \xi_{0s} - k_{0s} & p2_{0s} - \xi_{0s}^2 \end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1} \cdot \begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} -9.672 \times 10^{-5} \\ 2.944 \times 10^{-6} \end{pmatrix} N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 9.676 \times 10^{-5}\, N$$

Deuxième approximation

$$x_1(\alpha) := Re(z_1(\theta_0, \alpha)) \qquad y_1(\alpha) := Im(z_1(\theta_0, \alpha))$$

$$s\xi_1(\alpha) := \int_0^{\alpha} x_1(\alpha') \cdot d\sigma(\alpha')\, d\alpha' \qquad \xi_{1s} := \frac{1}{L} \cdot s\xi_1(\psi_0) \qquad \xi_{1s} = 0.198\, mm$$

$$s\eta_1(\alpha) := \int_0^{\alpha} y_1(\alpha') \cdot d\sigma(\alpha')\, d\alpha' \qquad \eta_{1s} := \frac{1}{L} \cdot s\eta_1(\psi_0) \qquad \eta_{1s} = 0.037\, mm$$

$$sp2_1(\alpha) := \int_0^{\alpha} x_1(\alpha')^2 \cdot d\sigma(\alpha')\, d\alpha' \qquad p2_1 := \frac{1}{L} \cdot sp2_1(\psi_0) \qquad p2_1 = 1.207\, mm^2$$

$$sq2_1(\alpha) := \int_0^{\alpha} y_1(\alpha')^2 \cdot d\sigma(\alpha')\, d\alpha' \qquad q2_1 := \frac{1}{L} \cdot sq2_1(\psi_0) \qquad q2_1 = 1.168\, mm^2$$

$$sk_1(\alpha) := \int_0^{\alpha} x_1(\alpha') \cdot y_1(\alpha') \cdot d\sigma(\alpha')\, d\alpha' \qquad k_1 := \frac{1}{L} \cdot sk_1(\psi_0) \qquad k_1 = 0.027\, mm^2$$

$$\mathbf{S_{1V}} := \frac{L}{E \cdot I_{33}} \cdot \begin{pmatrix} -y_1(\psi_0) \cdot \eta_{1s} + q2_1 & y_1(\psi_0) \cdot \xi_{1s} - k_1 \\ x_1(\psi_0) \cdot \eta_{1s} - k_1 & -x_1(\psi_0) \cdot \xi_{1s} + p2_1 \end{pmatrix}$$

$$\mathbf{R'} := \mathbf{S_{1V}}^{-1} \cdot \begin{pmatrix} x_V(\theta_0) - x_1(\psi_0) \\ y_V(\theta_0) - y_1(\psi_0) \end{pmatrix} \qquad \mathbf{R'} = \begin{pmatrix} -1.128 \times 10^{-4} \\ 1.873 \times 10^{-6} \end{pmatrix} N \qquad \left|\mathbf{R'}\right| = 1.128 \times 10^{-4}\, N$$

Contrôle des déplacements au point d'attache

$$\mathbf{\Delta z_V} := \mathbf{S_{1V}} \cdot \mathbf{R'} \qquad \Delta z_{1V} := \mathbf{\Delta z_V}_0 + i \cdot \mathbf{\Delta z_V}_1 \qquad z_{2V} := z_1(\theta_0, \psi_0) + \Delta z_{1V}$$

$$Re(z_{2V}) - x_V(\theta_0) = 0\, mm \qquad Im(z_{2V}) - y_V(\theta_0) = 0\, mm$$

Perturbation de période - spiral non déformé en position de repos

$$\sigma 2 := \frac{1}{L}\cdot\int_{0}^{\psi_0} z_0(\alpha)\cdot\overline{z_0(\alpha)}\cdot d\sigma(\alpha)\, d\alpha \qquad \sigma 2 = 2.705\, mm^2$$

$$X(\theta) := \frac{\left(\left|\Delta_1(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta}X(\theta)$$

$$\theta(\varphi) := \theta_0\cdot\cos(\varphi) \qquad Delta(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_{0}^{2\cdot\pi}\gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\, d\varphi$$

$$\mu(\theta_0) := -86400\cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = 73.078} \qquad \boxed{\mu(220\cdot deg) = 65.977} \qquad \boxed{\mu(300\cdot deg) = 77.062}$$